Biostats 270: HW 2

Andrew Holbrook

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For the following, please include code and output in a single pdf file. Assignment due by the beginning of class on Thursday 5/2.

1. We wish to simulate 1-dimensional Brownian motions for times $0 \leq t \leq 10$.

   (a) Letting $h = 0.01$, simulate 10 independent 1-dimensional Brownian motions for times $0 \leq t \leq 10$ using the discretization

   $$x(t+h) = x(t) + \sqrt{2h} z_t \,, \quad z_t \sim N(0,1)$$

   and **plot** each sample path using a different color.

   (b) For a Brownian motion $x(\cdot)$, the covariance of the Gaussian r.v.s $x(s)$ and $x(t)$ at two different times s and t is

   $$\text{cov}(x(s), x(t)) = \min(s,t) \,.$$

   Use this fact to simulate 10 independent Brownian motions for times $0 \leq t \leq 10$ using the grid points pertaining to the above discretization. **Plot** each sample path using a different color.

2. We wish to simulate Brownian motions on the interval $[0,1]$.

   (a) Use Euler-Maruyama with stepsizes of $h \in (0.1, 0.01, 0.001, 0.0001)$ to simulate 20 independent Brownian motions each. For each stepsize, how long does it take to generate all 20? **Plot** the sample paths.

   (b) Use the KL expansion with with $p \in (5, 10, 20, 40)$ eigenfunctions to simulate 20 independent Brownian motions each. For each number of eigenfunctions, how long does it take to generate all 20? **Plot** the sample paths.

   (c) Use the explicit form of the Brownian motion covariance function to obtain covariance matrices on uniform partitions of $[0,1]$ with 100, 1000, and 10,000 grid points. Use these covariance matrices to simulate 20 independent Brownian motions each. For each partition size, how long does it take to generate all 20? **Plot** the sample paths.

3. We wish to simulate Brownian bridges on the interval $[0,1]$.

(a) Use Euler-Maruyama with stepsizes of $h \in (0.1, 0.01, 0.001, 0.0001)$ to simulate 20 independent Brownian bridges each. For each stepsize, how long does it take to generate all 20? **Plot** the sample paths.

(b) Use the KL expansion with with $p \in (5, 10, 20, 40)$ eigenfunctions to simulate 20 independent Brownian bridges each. For each number of eigenfunctions, how long does it take to generate all 20? **Plot** the sample paths.

(c) Use the explicit form of the Brownian bridge covariance function to obtain covariance matrices on uniform partitions of $[0, 1]$ with 100, 1000, and 10,000 grid points. Use these covariance matrices to simulate 20 independent Brownian bridges each. For each partition size, how long does it take to generate all 20? **Plot** the sample paths.

4. We wish to draw samples from the 1-dimensional standard normal distribution. Use the 1D Fokker-Plank equation to derive the SDE with solution that leaves the standard normal target invariant. What is the name of this process? How does the parameterization of this process change when we change the variance of the Gaussian target?